AP Calculus AB
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Worksheet 4 - Rational Functions
Wednesday, November 2, 2022

Name:

A rational function is a function of the form

$$
f(x)=\frac{g(x)}{h(x)},
$$

where $g(x)$ and $h(x)$ are polynomials.
A rational function is in lowest form if the numerator and the denominator have no common complex zeros. Assume that $f(x)=g(x) / h(x)$ is a rational function in lowest form.

The degree of $f(x)$ is $\max \{\operatorname{deg}(g), \operatorname{deg}(h)\}$.
The zeros of $f(x)$ are the zeros of $g(x)$; that is, they are the solutions to $g(x)=0$.
The poles of $f(x)$ the zeros of $h(x)$; that is, they are the solutions to $h(x)=0$.
The $y$-intercept of $f(x)$ is the point $(0, f(0))$.
The $x$-intercepts of $f(x)$ are the points $(z, 0)$, where $z$ is a real zero of $f(x)$.
The vertical asymptotes of $f(x)$ are the lines $x=p$, where $p$ is a real pole of $f(x)$.
The polynomial asymptote of $f(x)$ is the polynomial equation $y=q(x)$, where $q(x)$ is the quotient when $g(x)$ is divided by $h(x)$ using polynomial division.


Rational Function: $f(x)=\frac{6}{x-2}$

## Degree:

## Zeros:

## Poles:

$y$-intercept:
$x$-intercepts:

## Vertical Asymptotes:

## Polynomial Asymptote:



Rational Function: $f(x)=\frac{4 x+2}{3 x-6}$

## Degree:

Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:


Rational Function: $f(x)=\frac{x-5}{x^{2}+x-6}$
Degree:
Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:


Rational Function: $f(x)=\frac{x^{2}-x-2}{x-2}$
Degree:
Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:


Rational Function: $f(x)=\frac{x^{2}-49}{x^{2}-25}$
Degree:
Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:


Rational Function: $f(x)=\frac{x^{3}-x}{x^{2}-9}$
Degree:
Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:


Rational Function: $f(x)=\frac{x^{2}-25}{x^{3}-3 x^{2}-4 x+12}$
Degree:
Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:


Rational Function: $f(x)=\frac{x^{3}-7 x+6}{x+1}$
Degree:
Zeros:
Poles:
$y$-intercept:
$x$-intercepts:
Vertical Asymptotes:
Polynomial Asymptote:

